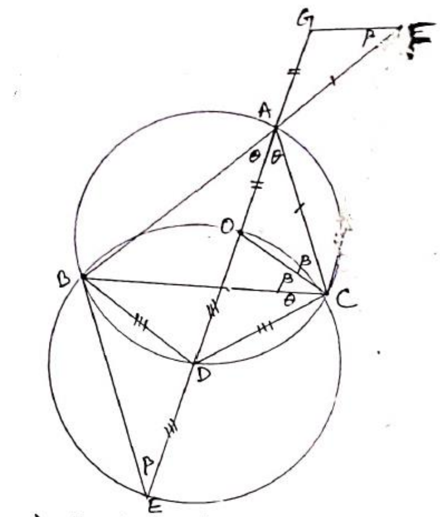
01.03.2024 - Cash Award Math Rider III Prize Winner Mr.Hrudanada Bhoi's Solution

Given :
'O' is the incentre of ΔABC, OA = AG, AC= AF, OD = DE

Claim :
BEFG is concyclic



Construction :
$\overline{BD}, \overline{CD}, \overline{OC}$ constructed.

Proof:

O is the incentre of $\Delta ABC \Longrightarrow$ m$\angle OAB = m\angle OAC$
But $m\angle OAB = m\angle GAF$ (opposite angle) $\Longrightarrow$ m$\angle OAC = m\angle GAF$
Now, in ΔOAC& ΔGAF we have $m\angle OAC$ = $m\angle GAF$,
OA= AG and AC = AF $\Rightarrow \Delta OAC \cong \Delta GAF$ (S-A-S)
$\Longrightarrow$ m$\angle OCA = m\angle AFG$ But m$\angle OCA = m\angle OCB$ as 'O' is incentre
Hence m$\angle AFG = m\angle OCB$ ---------------------------- (1)
Since m$\angle DAB = m\angle DAC$ we have BD=CD
Let m$\angle BAD = m\angle CAD = \theta$ m$\angle BCD = \theta$ (Angle inscribed in the same chord)
m$\angle OCA = m\angle OCB = \beta \Longrightarrow m\angle AFG = \beta$
Now the exterior angle m$\angle COD = (\theta + \beta)$
m$\angle OCD = m\angle OCB + m\angle BCD = (\beta + \theta) = (\theta + \beta)$
$\Longrightarrow m\angle COD = m\angle OCD \Longrightarrow DO = DC$
$\Longrightarrow DO = DC = DB = DE\ (As\ DO = DE\ given)$
Hence we can construct a circle with center.
D and diameter $\overline{OE}$, passing through B & C.
$\Longrightarrow m\angle OEB = m\angle OCB = \beta$
$\Longrightarrow m\angle GEB = m\angle GFB = \beta$
$\Longrightarrow$ BEFG are concyclic . -------------------- Proved

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